Kangaroo Math Competition, GeoGebra and Inclusive Education: teaching experiences based on Olympiad Problems use

Concurso Canguro de Matemáticas, GeoGebra y Educación Inclusiva: experiencia didáctica basada en Problemas de Olimpiada

Paulo Vitor da Silva Santiago [1]

Renata Teófilo de Sousa [2]

Francisco Régis Vieira Alves [3]

Abstract

This article presents a teaching experience that encourages adherence to the use of Olympiad Problems in the classroom, aiming at learning and inclusion of the deaf. In the particular case of this work, we suggest two statements present in the questions of the Kangaroo Math Competition Brazil 2023. The objective of this work is to present a teaching experience on the themes percentage and area of plane figures associated with Geometry, through two problems of the Kangaroo Math Competition with the contribution of GeoGebra. For this, we used the Didactic Engineering as a research methodology. The didactic situations were elaborated based on the Theory of Didactic Situations and from the concept of Olympic Didactic Situation and were structured with GeoGebra. The association of Olympic Problems to GeoGebra enabled students to build knowledge, from a visual perception and manipulation of the software, as well as the inclusion of deaf students, providing him with the development of mathematical knowledge.

[1] Licenciatura em Matemática, com especialização em Matemática. Doutorando em Ensino (UFC/CE). Secretaria de Educação do Ceará, Brasil. Correio eletrónico: paulovitor.paulocds@gmail.com. ORCID: https://orcid.org/0000-0002-6608-5452

[2] Especialista em Ensino da Matemática. Doutoranda em Ensino (IFCE/CE). Secretaria de Educação do Ceará, Brasil. Correio eletrónico: rtsnaty@gmail.com. ORCID: https://orcid.org/0000-0001-5507-2691

[3] Doutor em Educação pela Universidade Federal do Ceará, Bolsista de produtividade do CNPq – PQ2. Instituto Federal de Educação, Ciência e Tecnologia do Ceará, Brasil. Correio eletrónico: fregis@ifce.edu.br. ORCID: https://orcid.org/0000-0003-3710-1561

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Thus, the results obtained were positive, in addition to providing readers with a reflection on the teaching of mathematics, the use of Olympic problems and software for its teaching, as well as the inclusion of deaf students and their mathematical learning.

Keywords: GeoGebra, didactic engineering, kangaroo math contest, math olympiad problems

Resumen

Este artículo presenta una experiencia pedagógica que fomenta la adhesión al uso de los Problemas de la Olimpiada en el aula, teniendo como objetivo el aprendizaje y la inclusión de los sordos. En el caso particular de este trabajo, sugerimos dos enunciados presentes en las preguntas del Concurso Canguro de Matemáticas Brasil 2023. El objetivo de este trabajo es presentar una experiencia didáctica sobre los temas porcentaje y área de figuras planas asociadas a la Geometría, a través de dos problemas del Concurso Canguro de Matemáticas con la contribución de GeoGebra. Para ello, utilizamos la Ingeniería Didáctica como metodología de investigación. Las situaciones didácticas se elaboraron a partir de la Teoría de las Situaciones Didácticas y del concepto de Situación Didáctica Olímpica y se estructuraron con GeoGebra. La asociación de Problemas Olímpicos a GeoGebra permitió a los alumnos construir conocimiento, a partir de una percepción visual y manipulación del software, así como la inclusión de alumnos sordos, proporcionándole el desarrollo del conocimiento matemático. Así, los resultados obtenidos fueron positivos, además de aportar a los lectores una reflexión sobre la enseñanza de las matemáticas, el uso de problemas olímpicos y software para su enseñanza, así como la inclusión de alumnos sordos y su aprendizaje matemático.

Palabras clave: geogebra, ingeniería didáctica, concurso canguro de matemáticas, problemas de la olimpiada matemática

Introduction

The Mathematics Olympiads are present in the teaching and learning of Brazilian public and private institutions, as they address content that arouses the student's interest, encourages curiosity, and provides a motivating education, allowing the structuring of new mathematical knowledge. Competitions become more challenging for learning, in addition to bringing with them an overcoming character, when there are deaf students in this context (Mendes & Araújo, 2015). Inclusive education related to mathematics working in Olympic training is a universe with little research, which is a differential in this work.

This article presents Didactic Engineering (DE) with Plane Geometry content associated with Inclusive Education, based on the development of questions from the Kangaroo Math Contest Brazil. According to Almouloud (2007), DE is a research methodology based on an experimental scheme, which is present in didactic applications in the classroom, namely, in the elaboration, execution, observation and analysis of the teaching process, confronting the internal validation of a priori and a posteriori analysis.

The guiding question of this research was: how to insert the potentialities of GeoGebra in Mathematics Teaching, to promote a dynamic and inclusive learning of Plane Geometry with olympic problems? The teaching of Plane Geometry in an Olympic way is instigating, as it allows the work of the contents developed in the mathematics discipline, through the analysis of problems related to the practice. Thus, the objective of this work is to present a teaching experience on the themes percentage and area of plane figures associated with Geometry, through two problems of the Kangaroo Math Contest with the contribution of GeoGebra.

In this sense, we considered the context of Mathematics Teaching and Inclusive Education, bringing an experience in this perspective with the inclusion of a deaf student among the other participants. In order to contribute to the progress of the teaching and learning process, we use GeoGebra, as it is considered that "the presence of a certain technology can induce profound changes in the way of organizing teaching" (Kenski, 2012, p. 44).

Therefore, there are geometric constructions created in GeoGebra, Leivas et al., (2018), as a support for this learning process. It should be noted that GeoGebra is a virtual platform, application, and software, developed by Markus Hohenwarter in 2002, as part of his master's research at the University of Salzburg, Austria. From 2006, GeoGebra was funded by the Ministry of Education of this country, aiming to keep the software freely available for use in Mathematics Education in educational institutions and universities (Hohenwarter & Preiner, 2007; Hohenwarter, 2014).

In the following topics, we bring aspects about DE and its phases, structuring the epistemic-mathematical field of the object of study based on Olympic problems. Then, we present two Olympic didactic situations for their teaching, ending with the results and discussion of the lived experience and considerations of the authors.

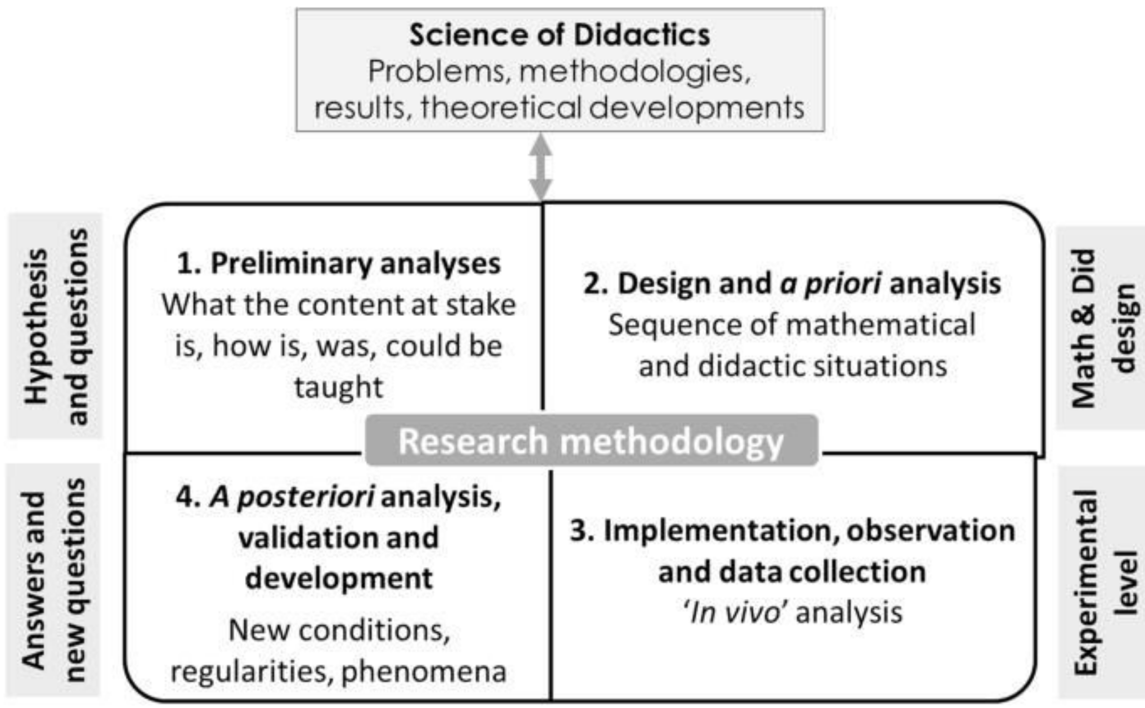
Development

Didactic Engineering (ED)

The nomenclature Didactic Engineering was created in the midst of discussions on Mathematics Didactics (DM) during the 1980s in France. The intention is to associate the teacher's work with the work of the engineer, whose production in the field requires basic, essential, and scientific knowledge, but also to develop solutions for practical problems, for which there is no prior theory and/or applications (Artigue, 1994; 1996). Thus, it is understood that DE provides support for structuring teaching sessions considering mathematical and didactic knowledge and highlighting the importance of research by the teacher.

The DE was created based on two questions: (a) the links between action in the education system and the research to be developed in the field and, (b) the environment for didactic constructions between research methodologies (Almouloud, 2007). “It can be presented as a research methodology structured in distinct intertwined phases with the double aim of studying didactic phenomena and developing new educational proposals” (Artigue 1990, apud Barquero & Bosch, 2015, p. 1). We seek to illustrate the DE scheme in Figure 1:

Figure 1. *Scheme of Didactic Engineering as a research methodology*



Source: Barquero & Bosch (2015)

In this way, the teaching practice in mathematics is developed through the investigation of the subjects. The DE stages guide the development of didactic proposals for teaching, based on the association between theoretical and practical knowledge. The idea of this study is to show microengineering, which takes place within the classroom, through the use of digital technology associated with the use of Olympic problems, in the context of inclusive education.

Artigue (1996) points out the stages of DE: (i) preliminary analysis; (ii) design and a priori analysis; (iii) experimentation, and (iv) a posteriori analysis and validation. Thus, throughout this study, we describe in a simplified way the development of a DE based on the objective proposed in the work.

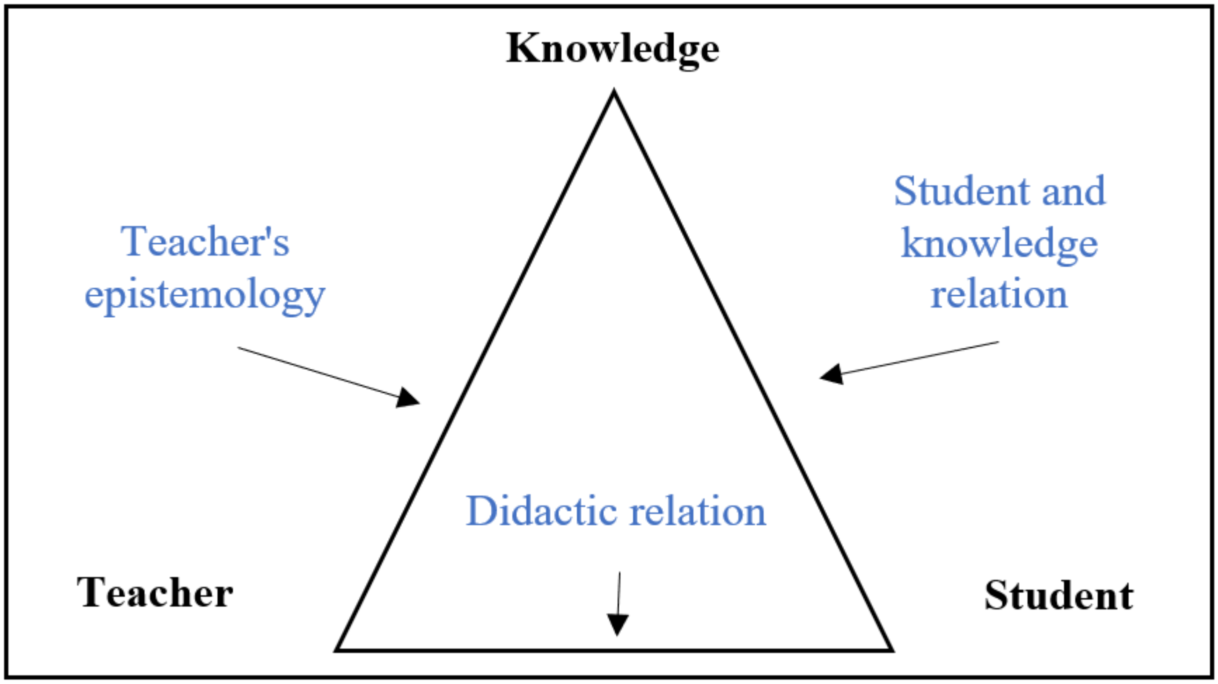
Preliminary analysis

In this step, we describe the theoretical survey that provided subsidy to the study in question. So, we are talking here about the Theory of Didactic Situations, which underpins the elaborated didactic situations, associated with the concept of Olympic Didactic Situation, implemented by Alves (2020).

Theory of Didactic Situations (TDS) and Olympic Didactic Situation (ODS)

Alves (2021) reports that over the last decades of research in France, the Didactics of Mathematics has been widely discussed based on phenomenological observations that surround the teaching and learning process in mathematics. Given this perspective, one of the studies developed to understand the phenomenon of learning in mathematics was the Theory of Didactic Situations (TDS), based on Brousseau thesis (1986). The author talks prosaically about the theoretical-cognitive field based on the didactic triangle, in which he considers the teacher, the student and knowledge as vertices and takes as references the epistemic field in which each of these actors is involved (Figure 2):

Figure 2. *Didactic Triangle*



Source: Adapted from Brousseau (1986)

Thus, TDS as a teaching theory, provides a contribution to teachers for the elaboration of didactic methods and the creation of a means (milieu) that promotes student learning, which is a protagonist in the construction of their knowledge.

The TDS allows the teacher to observe the classroom and the relationship between previous mathematical knowledge and the development of new knowledge by the students, from the promotion of a fertile discussion environment, through a process that is delineated by phases and/ or dialectic – action, formulation, validation, and institutionalization. These dialectics can be clearly observed during the discussions held between the students and the milieu and, in the last one, between students and the teacher (Brousseau, 2008).

Didactic knowledge interferes in students' discussions in the classroom, forming a range of symbols that build, structure, and justify validity relationships (Brousseau, 1986). The Table 1 shows the dialectics of TDS, according to Brousseau (1986), associated with some characteristics of cognitive thinking demonstrated by the student, as described by Alves (2021), where the author emphasizes that these are situations that need attention from the teacher of mathematics:

Table 1. *Description of the typology of cognitive thinking mobilized in the TDS phases*

Phases of TDS	**Characteristics of cognitive thinking**
Situation of Action	Without distinction or priority, in order to encourage training and mobilize the greatest possible number of representatives involved in the problem situation.
Situation of Formulation	At first, the students applied the record with the policy formulated and implemented, but they were unable to prove it. This phase awakens the symbology that is linked to the instrumentalization functions of the problems.
Situation of Validation	Actions in the form of partially or fully proven inferences, logical-mathematical reasons for confirming or rejecting a specific question.
Situation of Institucionalization	The reasoning is seen as a conceptual tool that must be incorporated and used as cultural knowledge to assist in later situations.

Source: Elaborated by the authors

To outline the Kangaroo problems in our discussion, we insert the concepts from the perspective of the Theory of Didactic Situations (TDS) together with the Olympic Problems (OP), which result in what Alves (2020) calls the Olympic Didactic Situation (ODS), being a term that still finds gaps in the literature.

The SDO was conceived and defined based on the concept of didactic situation, which derives from the TSD, and which generally refers to the teaching of mathematics and learning phenomena. In an analytical point, the notation of the mnemonic of Alves (2021), describes the composition of an Olympic Didactic Situation (ODS) by the characteristic equation ODS = OP + TDS, in which the TDS is the guiding teaching theory and OP is the Olympic problem on which the structuring of the didactic situation is based.

According to the TDS, which is guided by a specific context, in this case it would be mathematical knowledge applied to the resolution of Olympic issues, and an educational system. This context aims to allow students (potential competitors) to be immersed in an Olympic competition environment. This can stimulate knowledge, based on group discussion and scientific debate of mathematical concepts, in addition to the exploration of a series of problems characteristic of tests from different Mathematics Olympiads.

The OP is a set of mathematical problem-situations, which is structured in approaches extracted from a competition or marathon, involving methods and characteristics of the action of the subject's knowledge in solving the question, reaching defined goals in each competition, and which may have his approach outlined by the math teacher (Alves, 2021).

In this sense, the technological resources present in teaching can play a crucial role in changing teachers' pedagogical practice (Artigue, 2013). In this way, we can understand that the mnemonics arising from the application and development of OPs associated with the use of digital technology can present a modeling for geometric objects, and, simply, promote the relationships between teacher, student, and knowledge in a natural way, even in the face of other complex mathematical structures presented in the classroom.

Here, in this preliminary analysis, we are based on two premises: I) an introduction about the epistemology of teaching Plane Geometry in High School, and; II) an analysis of the textbooks of the Mathematics Olympiads regarding the content of Plane Geometry. Based on premise I, we carried out a survey of works, where we found authors such as Cury (2019), who emphasizes the importance of including textbooks in Basic Education according to Depth Hermeneutics, and which is based, consequently, on a socio-historical and formal analysis of Plane Geometry content. The authors Teixeira & Mussato (2020), when dealing with the concept of Didactic Sequence together with Geometric Solids with faces flattened through GeoGebra in the early years of Elementary School, show us that this didactic resource makes math classes more attractive. In this way, the software allows the creation of possibilities for dynamic classes for students.

In premise II, we looked for materials on the subject in the books made available by the Brazilian Mathematical Olympiad of Public Schools (OBMEP, acronym in Portuguese), in the sections that bring problems related to the content of Plane Geometry. The selected problems are also based on the content matrix of the Kangaroo Mathematical Olympiad Brazil and were modeled in the GeoGebra software, in order to support students and teachers in the (re)adaptation of Olympic questions.

Among the analyzed books, we highlight the work "An Introduction to Geometric Constructions" (Wagner, 2015), which presents Geometry problems that have already been included in this competition model. It was also observed how the contents are worked with the students, following the traditional standards of teaching and learning

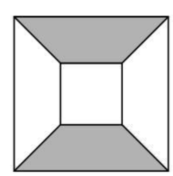
A priori analysis

In a priori analysis and the design of the didactic situation, two problem situations were considered and planned, which we treat as ODS, structured from data from two OPs extracted from the Kangaroo Test (2023) for High School students (Level J), dealing with concepts of Plane Geometry. The two ODS were developed based on TDS principles and structured with GeoGebra. The proposed problems address the percentage and area of plane figures, in addition to other basic concepts that we hope will be presented as prior knowledge by the students, such as basic operations. The first ODS can be illustrated in Figure 3:

Figure 3. *First ODS*

04. A large square of side-length 10 cm contains a smaller square of side-length 4 cm, as shown in the diagram. The corresponding sides of the two squares are parallel. What percentage of the large square is shaded?

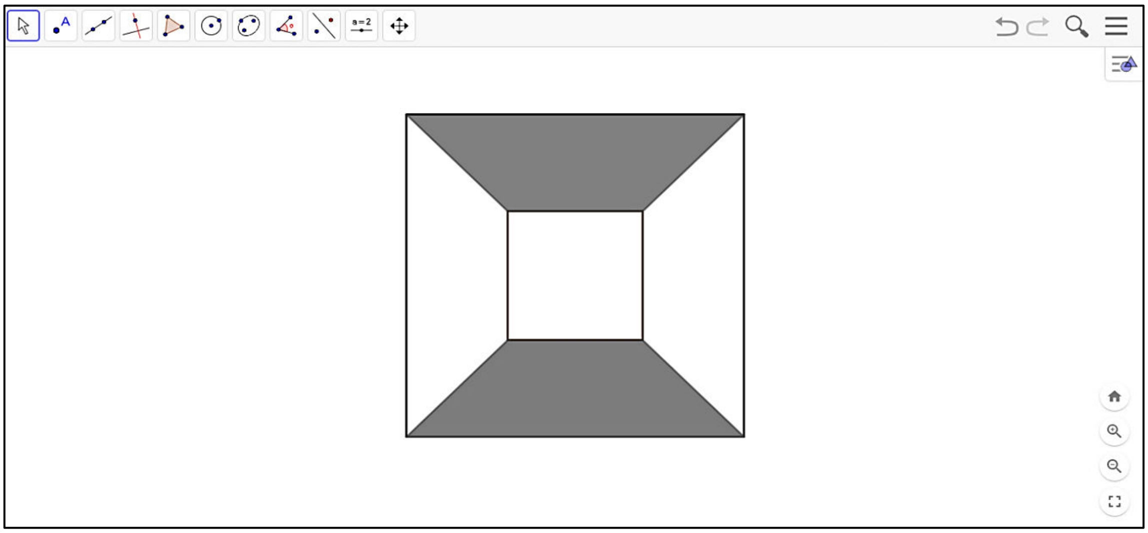
(A) 25% **(B)** 30% **(C)** 40% **(D)** 42% **(E)** 45%



Source: Kangaroo Math Contest (2023)

Faced with the statement of the problem, we developed a construction in GeoGebra to support the students (Figure 4). Then, we present a QR code (Figure 5) so that the student has access to the construction using the GeoGebra Suite application for smartphones or tablets:

Figure 4. *Building the ODS 1 in GeoGebra*



Source: Elaborated by the authors

Figure 5. *Code that allows access to the ODS 1*

Source: Elaborated by the authors

Below, we present the second ODS, which is another problem of the same test, which was developed with the students (Figure 6):

Figure 6. *First ODS*

06. The large rectangle in the diagram is divided into 30 equal squares, as shown. The perimeter of the shaded region is 240 cm. What is the area of the rectangle?

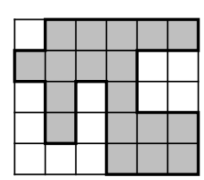
(A) 480 cm^2

(B) 750 cm^2

(C) 1080 cm^2

(D) 1920 cm^2

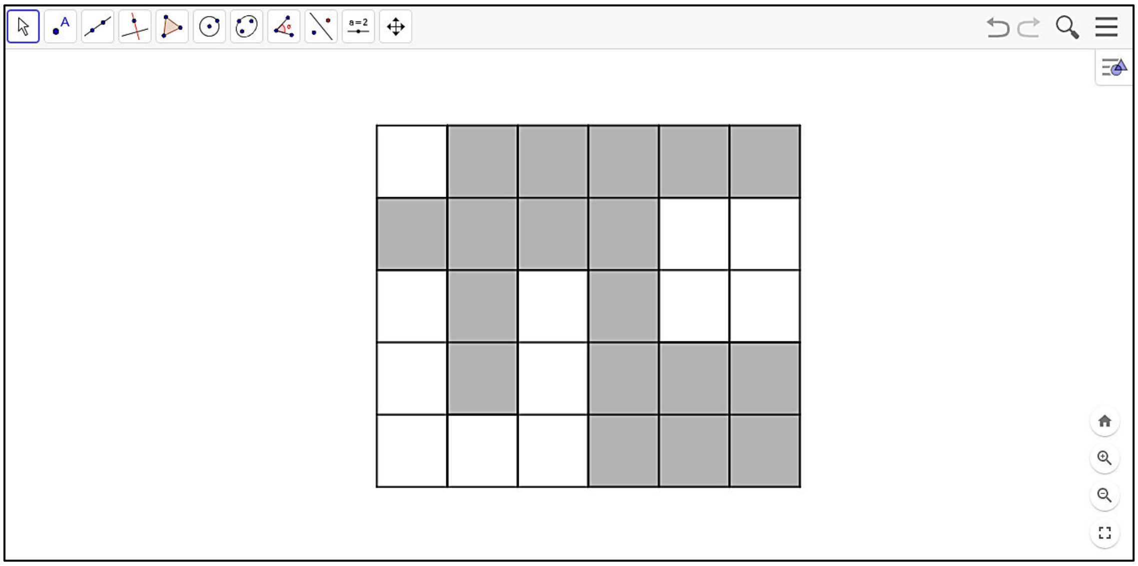
(E) 2430 cm^2



Source: Kangaroo Math Contest (2023)

We also provide a construction (Figure 7) for a better development of the student's reasoning, as well as its QR code (Figure 8), to be accessed by the GeoGebra application:

Figure 7. *Building the ODS 2 in GeoGebra*



Source: Elaborated by the authors

Figure 8. *Code that allows access to the ODS 2*

Source: Elaborated by the authors

Given the brevity of the study and its limitation in terms of length, we move on to the next section on the experimentation phase of the structured DE, where we describe the design of the experiment.

Methodology

Experimentation

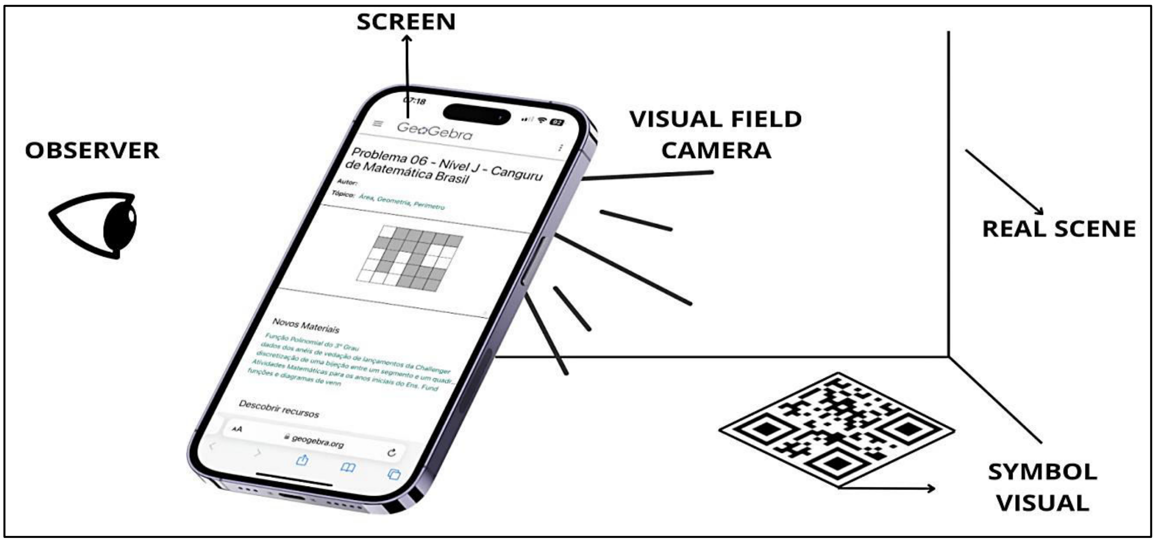
The work had DE as methodology, based on Artigue (1988) and followed a course based on the concept of didactic sequence. According to Zabala (1998), the didactic sequence is divided into organized activities, defined, and outlined for the development of learning, with specific educational objectives to be achieved by students. The experimentation stage of this DE had the participation of a group of 16 students who are preparing for tests in the Mathematics Olympics at the João de Araújo Carneiro Full-Time Secondary School, located in the rural area of the city of Quixeramobim, Ceará, Brazil, and who participate in the Intensive Training Olympic Poles (POTI, acronym in Portuguese) as participants in Olympic training.

Four meetings were held, in four classes lasting 50 minutes each. Data collection was carried out from records of observations, their speeches and questions from students, audio recording, photographic records and calculations performed by them. The classes were held in face-to-face format and the data were analyzed according to the TDS phases. The selection criterion for these students was to be a participant in the POTI, as this training program aids both teachers and students to study mathematics at an Olympic level.

It is worth mentioning that one of the students observed is deaf and that he attended the program meetings, receiving support from colleagues in mathematics training classes for the Olympics. The school does not have a sign language interpreter and communication between the other students and the deaf student and between the teacher and the deaf student takes place through simple gestures. The learning of mathematics by the deaf student showed a good performance, even in the face of the difficulties faced.

Students were identified in the survey as P1, P2, P3, ..., P16, respecting the requests of the Research Ethics Committee and preserving their identities throughout the study application process. The Olympic class in question is well diversified and, as stated, has a large number of maths literate students, including an Olympic deaf student. For this research, students were guided by their maths teacher to download the GeoGebra app on their mobile phones. The first interaction was to recognise and familiarise with the app and the basic commands, such as viewing objects in the QR-Code, moving objects or sliding control (Figure 9). The intention was for them to see the geometric objects used to calculate the percentage and area of plane figures.

Figure 9. *Simulation of GeoGebra app on mobile phone with QR-Code*



Source: Elaborated by the authors

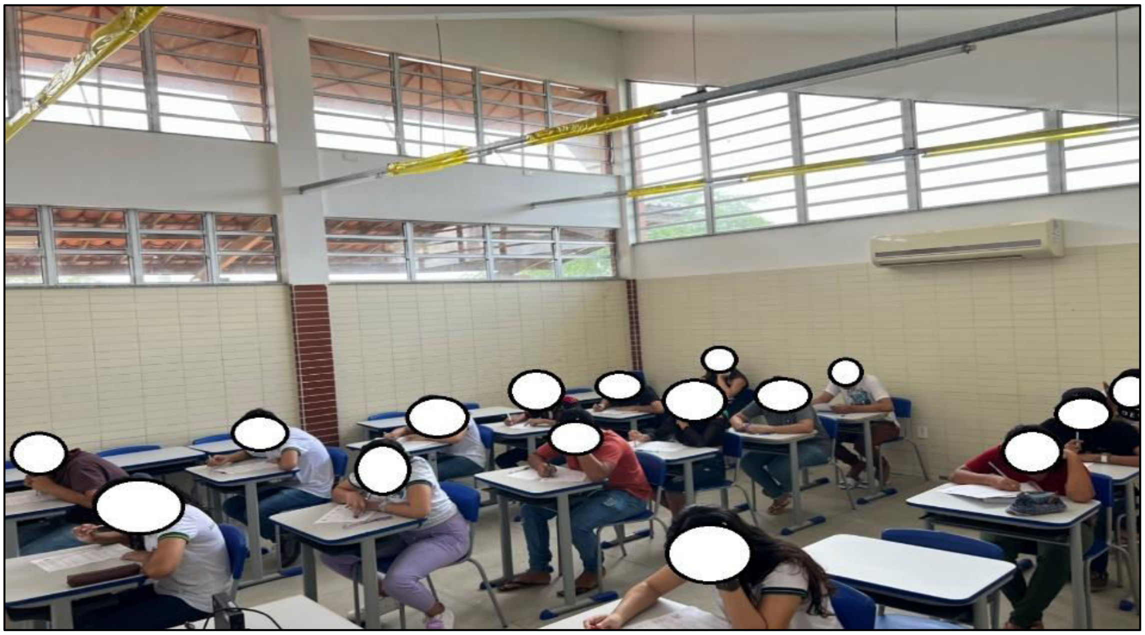
Osypova & Tatochenko (2021, p. 183) "the environment provides the organization of the process of teaching natural sciences and mathematics the design of dynamic graphic objects and conducting research using augmented reality". In the a posteriori analysis and (internal) validation, the analysis of the data collected in the Olympic application were carried out with the objective of listing the verification of the OPs solved by the students and their analyzes regarding the use in the mathematics laboratory.

A posteriori analysis and internal validation

Regarding the ODSs worked on, the teacher made them available in printed format for the class, allowing students to cross out on paper and draft their conjectures. In addition, the teacher presented the images of the constructions carried out in GeoGebra in a multimedia projector, as a way to facilitate the discussion among the students in the TDS formulation phase.

In Figure 10 we have a photographic record of the students in the classroom, at the time of the face-to-face meeting:

Figure 10. *Initial contact with the two Olympic problems*



Source: Survey data (2023)

In Figure 10 we have the students' first contact with the statement of the two ODSs and their information, so that they can begin their resolutions and attempts. After this first contact, the students had the opportunity to see the two constructions proposed by the mathematics teacher in a projected way, with the objective of developing an initial reasoning and strategies for their resolutions, since not all students had a smartphone or tablet. So, the teacher helped the students without access to technology by designing these ODSs, so that, in a collaborative way with the other students who had a cell phone, they could develop and understand the proposed situations.

Initially, the professor, researcher and mediator of the meeting asked some questions to instigate the students, such as: “what is the most relevant information for you to develop the solution to the problem?” The class read the ODS carefully, with the aim of identifying which ideas were proposed in the situation of action. They also observed each other's attempts at strategies to solve the two problems. The student identified as P11 presented the following description to the group:

P11: I read the statement three times to understand what was being presented as a solution to the question, to do some calculations until I reached the answer I wanted in view of the demonstration presented by the teacher with the construction in GeoGebra. So, I created the same figure to find the percentage per part, as if it were a test to find out what the final result of the Olympic math question would be.

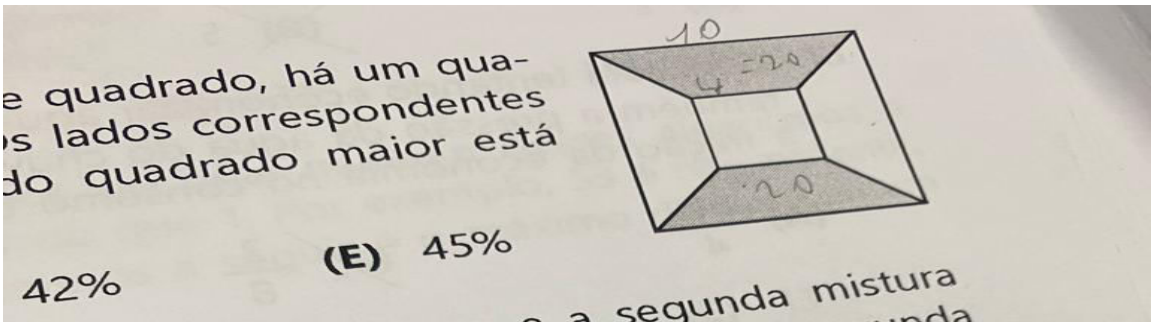
The teacher continued his questioning with the other students, including the deaf student, showing the expanded construction so that they could observe the ODSs. During the formulation situation, within the milieu established by the research professor, students P2 and P5 described their understanding as follows:

P2: I already solved the question in another way; I drew the larger and smaller squares. From there, I calculated the areas of each one, overlapping the segments of each diagonal. In this way, I was able to develop the percentage of the drawing.

P5: I drew the complete figure provided by the teacher and found the total percentage. Now, I will decrease the values of each area to develop the calculation of the percentage of the figure referring to the fourth question.

Each student presented different ideas to solve the two ODSs, but that somehow complemented each other with the question statement data, which embodied the second stage of the TSD, the situation of formulation. Here, ideas were generated by structuring intuitive cognitions pointed out by students, as stated by Alves (2020) in our preliminary analysis. These structures of thought during the TDS led the students to choose an alternative that could give real meaning to their resolution. We have this illustrated in Figure 11:

Figure 11. *Resolution of ODS 1*



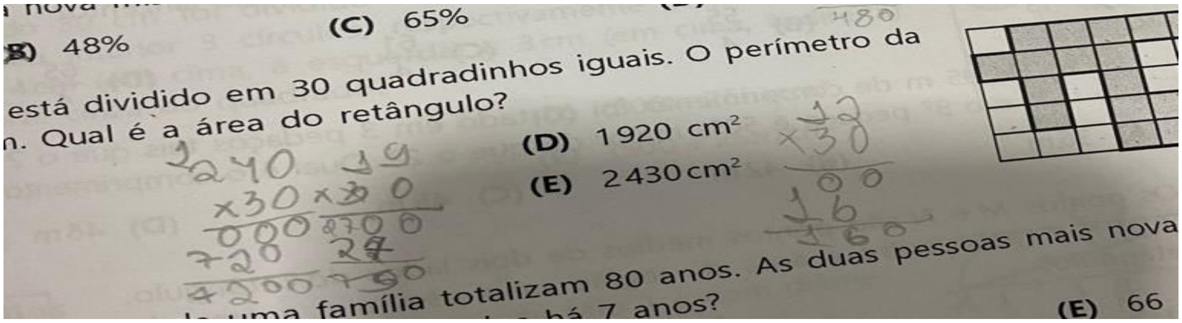
Source: Survey data (2023)

The indicated descriptions provoked another questioning among the students, based on the reasoning of P7, which indicates the situation of formulation of the ODS. In this case, another strategy related to the calculation of the percentage of the plane figure was used, which runs through the four trapezoids present in the statement of the question.

P7: The area of a plane figure is measured by its surface, which is equivalent to the sum of the measures of the sides of the equivalent square. The largest being 10 cm., and the smallest measuring 4 cm. Thus, I managed to have the areas of the larger side (10 cm x 10 cm = 100 cm²), the smaller side (4 cm x 4 cm = 16 cm²) and subtracting the two (100 cm² - 16 cm² = 84 cm²). In this way, finally we have the division of two areas that will be (84 cm² / 2 = 42 cm²).

Another resolution was described from the discussion between students P13 and P2. From this, student P2 was able to develop the mathematical calculation of ODS 2 (Figure 12) and show the class how he reached this conclusion:

Figure 12. *Description of ODS 2 by P2*



Source: Survey data (2023)

P2's discussion, as shown in Figure 12, brings the resolution of the SDO, configuring the beginning of the TSD validation situation as follows: the statement of the question demarcates the gray area of 240 cm2. In this way, it is understood that the perimeter is equal to the sum of all the external sides of the squares of the gray area:

P2: The value of 240 cm divided by 30 squares results in 8 cm of each square. Given this answer, the ratio of length (5 x 8 cm = 40 cm), width (6 x 8 cm = 48 cm) and area (40 cm x 48 cm = 1920 cm²) is performed, finding the total area of the rectangle.

During the class, the teacher was asked by the students if he could help during the didactic situation (first three phases of the TDS). The teacher, when mediating the situation, replied to the class that he could show whether the calculation method being used was adequate to resolve the issue. What helped the students, in fact, was the projection of images of the buildings on the multimedia projector.

The discussions presented relevant data for the final resolution of each ODS, but some students did not reach a formal response. That is, even seeing the construction in GeoGebra, they still had difficulties in developing the geometric thinking necessary for its solution. Others succeeded, including the deaf student, who showed characteristics of advanced geometric thinking and great mathematical potential.

Student P4's explanation regarding the area of ODS 2 shows that he did the total calculation without subtracting the squares painted in gray. In addition, this student showed another different idea of what is sought in solving the Olympic problem with the support of GeoGebra.

Students organized all relevant data developed and presented in the first three phases of the TSD, in formal mathematical language, seeking to align the results clearly and with solid arguments, with the aim of confirming the established hypotheses. Brousseau (1986) explains that the validation of the didactic situation brings a solution where the subjects establish the validity of the knowledge acquired about the problem.

The teacher accompanied the mathematical and logical-geometric reasoning of each student, resuming his role as mediator and encouraging the class to clarify any doubts. This moment was consolidated as the situation of institutionalization of the TDS, which, according to Brousseau (1986), is the stage in which the teacher formally presents mathematical knowledge, based on what was exposed and discussed by the students in the previous stages.

In this phase, some concepts were resumed and explained by the teacher, focusing on the percentage and area of plane figures, which allowed students to find other ways of thinking and solving each ODS presented. According to Santiago & Alves (2021), an OP allows the development of different resolution techniques, which go beyond the cognitive aspects of math questions worked on in textbooks.

The use of GeoGebra allowed students to intuitively perceive the characteristics and properties of each geometric shape presented, streamlining geometric thinking through demonstrations of solutions with sliders.

The ODS 1, which addressed the percentage associated with flat figures, was more difficult for students to interpret. However, ODS 2, which used the checkered grid associated with the areas, showed more accurate results. The researcher professor analyzed each test, the errors of signs, calculations, and descriptions of the students' resolutions, but given the brevity of the extension of this work, they are not presented here.

Finally, we understand that the constructions provided students with a way to understand the dynamics of the figures of each ODS. With GeoGebra, mathematical knowledge can be developed more effectively than with traditional teaching, especially geometric thinking, since the construction and manipulation possibilities stimulate the student's logical-mathematical reasoning, allowing their learning, including the possibility of its use with deaf students, as was the case. The participants' independence with the tool also encourages us to use it for future applications with other geometry topics related to Olympic exams.

Conclusions

In this work we use the concept of ODS, based on the Theory of Didactic Situations, developed from two problems arising from the test of the Kangaroo Mathematics Contest, in this year, 2023. The questions dealt with the concept of percentage and area of plane figures. The choice of these mathematical contents was due to their importance in other external assessments, not necessarily Olympic ones, and the need for their approach in the classroom, in addition to recurrent difficulties in learning these themes evidenced by the students.

In this way, we developed the two ODSs organized in Didactic Engineering, as a way of understanding these themes, structuring a teaching session, and developing it in a school environment. We also understand that the methodological model used allowed a better organization and data collection. The information collected in the experimentation during the

Olympic training of these students shows us that, even with all the difficulties, learning occurred, including the deaf student, who was successful in the development of ODSs, reaching the objective of the research.

DE enabled us to create an interactive environment in advance, based on the phenomenological understanding of learning these themes and the structuring of the didactic sequence based on the TDS phases. The students' discussions showed that they developed an understanding of mathematical concepts and, at the same time, returned to basic subjects, such as the study of areas of plane figures and percentage. Each student proceeded autonomously showing different solution possibilities and mathematical paths for solving the two ODSs.

GeoGebra as a support helped the teacher in transposing the two proposed OPs, enabling a dynamic and interactive milieu, and arousing the students’ interest. The projection of the developed buildings, their visualization and movement enabled the understanding of some resolving methods for the proposed situations, as well as the ability to develop other forms of resolution for these.

In addition, the adaptation of the class to the deaf student, their reception and inclusion, as well as the development of this student with the support of GeoGebra, showed us that it is possible to use technology to structure didactic models, in a perspective that associates the use of Olympic questions from mathematics to the context of Inclusive Education.

Finally, we make this experience available to mathematics teachers as a didactic model of teaching, based on the theories listed above, encouraging them to adopt postures different from traditional teaching in their practice, in order to search for other ways to develop mathematical knowledge in students.

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